Decision-Making for Faculty Recruitment using Intuitionistic Cubic Fuzzy Graphs

Uzma Ahmad[1], Areeba Maqbool[2] and Saira Hameed[3]

[1,2&3]Institute of Mathematics, University of the Punjab, Lahore, Pakistan.

Correspondence:

Uzma Ahmad: uzma.math@pu.edu.pk

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Uzma Ahmad (Corresponding Author), Institute of Mathematics, University of the Punjab, Lahore, Pakistan.
Email: uzma.math@pu.edu.pk

Areeba Maqbool, Institute of Mathematics, University of the Punjab, Lahore, Pakistan.

Saira Hameed, Institute of Mathematics, University of the Punjab, Lahore, Pakistan.

ABSTRACT

Multi-Criteria Decision Making (MCDM) models most often lack the ability to simultaneously account for the relational structure among criteria, hesitation and uncertainty in the human judgement, as shown by this paper. Therefore, this paper proposes a new approach Interpreting Intuitionistic Cubic Fuzzy Graphs (ICFG) Model using the Additive-Ratio Assessment (ARAS) Method, which uses an additive-ratio method. The resulting ICF-ARAS model provides the most logical method for creating a structured relational model of a decision-making element and includes interval degree membership, non-membership and hesitation. To demonstrate applicability of this new MCDM Methodology, this paper provides a case study on Faculty Recruitment, comparing 4 candidates on 4 criteria (qualifications, interview results, teaching experience and communication), the resulting model produced an identical ranking ($P_2 \geq P_1 \geq P_4 \geq P_3$) as established alternative methodologies (ICF-TOPSIS and ICF-WASPAS) while offering enhanced interpretability, computational simplicity, and relational transparency. The new approach provides an effective, transparent and flexible decision-support mechanism for selecting multi-faceted and uncertain candidates in higher education and elsewhere.

Keywords: Intuitionistic Cubic Fuzzy Graphs, Multi-Criteria Decision Making, ARAS Method, Faculty Recruitment, Fuzzy Graph Theory, Decision Support Systems

JEL Classification Codes: C44, I23, J45, M51

1. INTRODUCTION

Hiring the appropriate educator is one of the most important steps in the entire hiring process because the educator you choose will have a large impact on how effectively your students learn. When evaluating educators, schools must consider a variety of factors (such as educational qualifications, teaching experience, communication skills, classroom management, research contributions, and student evaluations). As each of these criteria involves some degree of subjectivity and different weights from experts, determining the best educator will often be a complex and uncertain task. To overcome this challenge, this paper introduces a new methodology that incorporates the Additive Ratio Assessment (ARAS) methodology and the cubic fuzzy graph (CFG) model to account for the uncertainty associated with expert evaluation of educator candidates. The proposed methodology combines qualitative and quantitative evaluations of candidates, thus allowing for an unbiased, fair, and transparent process for selecting the best educator for your school. A graph-theoretic approach is used to organize candidates and the evaluation criteria so that they can be systematically analyzed through an algorithmic framework that takes into account both the relationships among candidates and criteria as well as the uncertainties surrounding those relationships.

Our contribution includes a complete integration of ICFG's and ARAS's for modelling structured uncertainty and a complete mechanism for transparent utility-based ranking of alternatives that improves the interpretability of results. Lastly, a case study on an academic recruitment effort demonstrates the utility of these contributions and how they compare to established practices.

1.1. Significance and Novelty

Decision-making frameworks for HRM have seen an increase in focus on transparency and fairness in candidate selection, particularly in the academic sector. HRM applicant evaluation processes have historically relied on the subjective judgments of related experts which lead to varying degrees of bias. As such, MCDM models were developed to provide a more systematic approach to candidate evaluation that takes into account the many quantitative metrics used to measure candidates. Regardless of recent innovations in MCDM models, allowing for uncertainty in hiring expert evaluations has been an ongoing challenge for HRM decision-making models. This paper presents an ICFG based ARAS framework that incorporates uncertainty modelling within an established methodology of structured decision-making in HRM. By using a mathematical framework of HRM-relevant factors, the authors provide further support for continuing development of the literature around transparent, data-driven hiring within higher education and beyond.

Although recent studies have applied intuitionistic fuzzy and cubic fuzzy MCDM techniques to decision-making problems, most existing approaches either ignore relational structures among criteria or fail to adequately model hesitation and uncertainty simultaneously. Moreover, many methods rely on distance-based ranking, which reduces interpretability for real-world stakeholders. The present study addresses these gaps by combining ICFG with the ARAS method, enabling structured relational modeling and direct utility-based ranking.

1.2. Objective of the Study

This research aims to assist in faculty hiring through a transparent, sustainable, MCDM process using ARAS method and ICFG to represent an entity (candidate) in a transparent manner, provide for uncertainty, and capture the collection of criteria and candidate relationship interaction for each of the candidates. The outcome from implementing the model through the proposed methodology will provide for substantive increases in fairness, interpretability, and reliability of the decision-making process of faculty hiring within higher education institutions.

The rest of the article is organized into six sections. Section 2 provides preliminary information and Section 3 describes the ICF-ARAS algorithm. Section 4 shows how the framework can be applied in the context of selecting teachers. Section 5 discusses findings and provides a comparative analysis. Finally, Section 6 concludes the article with a discussion of the implications and future directions of this research.

2. LITERATURE REVIEW

Fuzzy graph (FG) theory was developed as a result of the integration of fuzzy set theory and graph theory. FG theory deals with situations in which the inherent vagueness of real-world systems cannot be adequately captured by crisp binary relationships. In fields where imprecision and uncertainty are inevitable, such systems frequently appear in broadcast communications, artificial intelligence, science and engineering, and neural networks. FG offer a versatile mathematical framework for expressing ambiguous relationships by permitting vertices and edges to have degrees of membership. Shi et al. (2024) provide a thorough summary of current advances in FGs.

Interval-valued fuzzy sets (IVFSs) extend classical fuzzy sets by replacing single membership values with intervals, thereby capturing higher levels of uncertainty. The fusion of IVFSs with graph theory was initially formulated by Hongmei and Lianhua (2009). Subsequently, Akram and Dudek (2011) introduced several algebraic operations on IVFGs, while Pal and Rashmanlou (2014) investigated structural properties of highly irregular IVFGs. To further enhance modeling capability, Jun et al. (2011) proposed cubic sets, which combine fuzzy sets and IVFSs to represent complex uncertainty patterns that cannot be handled by conventional fuzzy models alone. Building on this concept, Rashid, Yaqoob, Akram, and Gulistan (2018) introduced CFGs. After identifying limitations in the original definitions, Muhiuddin et al. (2020) provided revised and consistent formulations. Since then, several structural characteristics of CFGs have been examined, including connectivity and connectivity indices (Jun et al., 2011), regularity (Muhiuddin et al., 2022), bridges (Krishna et al., 2019), and planarity (Rao et al., 2024). Due to their enhanced flexibility, CFGs have been widely applied in modeling complex systems such as image processing, economic networks, traffic flow, and decision-support environments.

Another important extension is intuitionistic fuzzy sets (IFSs), introduced by Atanassov (1999), which characterize uncertainty using both membership and non-membership degrees. Parvathi and Karunambigai (2006) extended this framework to IFGs, allowing simultaneous representation of acceptance and rejection in network structures. Later, Ismayil and Ali (2014) proposed interval-valued intuitionistic fuzzy graphs to further accommodate imprecision. The concept was advanced by Muneeza and Abdullah (2020) through IFSs, integrating cubic and intuitionistic representations. This evolution led to the development of ICFGs in 2021, enabling richer modeling of uncertainty in graph-based systems. More recently, Fang et al. (2023) introduced planarity concepts for ICFGs, extending their applicability to complex topological and decision-making problems.

Parallel to these developments, MCDM methods have been extensively employed to evaluate alternatives involving multiple, often conflicting, criteria. MCDM supports decision-makers by incorporating both quantitative and qualitative factors with assigned importance weights. Popular approaches include the Analytic Hierarchy Process (AHP) (Mahad, Yusof, & Ismail, 2021), the Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) (Amudha et al., 2021), and the Weighted Aggregated Sum Product Assessment (WASPAS) method (Zavadskas et al., 2013). To handle vague and uncertain relationships among criteria and alternatives, Zavadskas, Turskis, and Vilutiene (2010) proposed the ARAS method, which was later extended into fuzzy environments by Turskis and Zavadskas (2010).

Motivated by the growing need to integrate advanced FG structures with decision-making frameworks, this study develops an intuitionistic cubic fuzzy ARAS (ICF-ARAS) method. The proposed approach embeds the ARAS technique within the environment of ICFGs, enabling more robust handling of uncertainty, hesitation, and interval information in complex decision scenarios. Consequently, the method provides an effective tool for practical applications where both structural relationships and multi-criteria evaluations must be addressed simultaneously.

3. PRELIMINARIES

Definition 3.1. A FS L on $X \neq \emptyset$ is prescribed by a membership function

$\Psi : X \to [0, 1]$,

It can be represented as

$L = \{(u_s, \Psi(u_s)) : u_s \in X\}$.

The support and support length of L are defined as $supp(L) = \{u_s \in X \mid \Psi(u_s) \neq 0\}$ and $s(L) = |supp(L)|$, respectively. The core and core length of L are defined as:

$core(L) = \{u_s \in X \mid \Psi(x) = 1\}$ and $c(L) = |core(L)|$,

respectively. The height of L is defined as $h(L) = \max\{\Psi(u_s) \mid u_s \in X\}$. The fuzzy set L is called normal if $h(L) = 1$.

Definition 3.2. A FG over $X \neq \emptyset$ is a pair (P, Q), where P and Q represent the fuzzy set FS on X and $X \times X$, respectively. It is prescribed by a membership functions $\Psi_p : X \to [0, 1]$ and $\Psi_Q : X \times X \to [0, 1]$, such that

$\Psi_Q(u_{s-1}, u_s) \leq \min\{\Psi_P(\upsilon_{s-1}), \Psi_P(\upsilon_s)\}, \forall \upsilon_{s-1}, \upsilon_s \in X\}$,

where Q is a fuzzy relation on P.

Example 3.3. Let $\mathbb{P}$ and $\mathbb{Q}$ be the FSs on $U = \{v, w, x\}$, and $U \times U$, respectively. The fuzzy membership values are given in Tables 1 & 2, respectively. The graphical representation of FG is shown in Figure 1.

Table 1: A FS $\mathbb{P}$ on U

U	***V***	***w***	***X***
$\mathbb{P}$	0.7	0.3	0.5

Source: Author's own

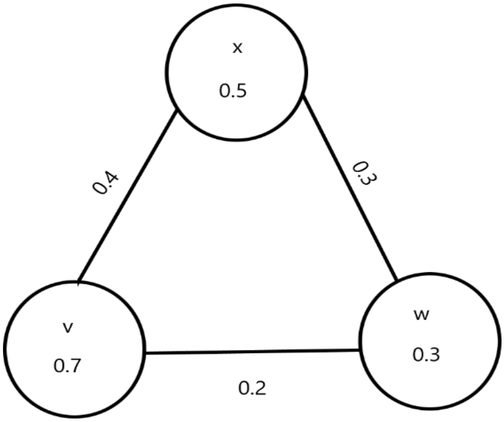
Table 2: **A FS $\mathbb{Q}$ on $U \times U$**

$N \subseteq U \times U$	***vw***	***Wx***	***Vx***

$\mathbb{Q}$	0.2	0.3	0.4

Source: Author's own

Figure 1: A FG on *U*



Source: Author's own

Definition 3.4. (Akram and Dudek, 2011) An IVFG R′ on X is a pair (P', Q'), such that $P' = [\Psi_{P'}^{-}, \Psi_{P'}^{+}]$ and $Q' = [\Psi_{Q'}^{-}, \Psi_{Q'}^{+}]$ are IVFSs on X and $X \times X$, respectively as Ψ_{P}': $X \rightarrow D\,[0, 1]$ and Ψ_{Q}' : $X \times X \rightarrow D\,[0, 1]$ so that $\forall \upsilon_{s-1}, \upsilon_s \in X$,

$$\Psi_{Q'}^{-}(\upsilon_{s-1}, \upsilon_s) \leq min\,\{\Psi_{P'}^{-}(\upsilon_{s-1}), \Psi_{P'}^{-}(\upsilon_s)\}, \forall \upsilon_{s-1}, \upsilon_s \in X,$$

$$\Psi_{Q'}^{+}(\upsilon_{s-1}, \upsilon_s) \leq min\,\{\Psi_{P'}^{+}(\upsilon_{s-1}), \Psi_{P'}^{+}(\upsilon_s)\}, \forall \upsilon_{s-1}, \upsilon_s \in X,$$

where Q' is a fuzzy relation on P'.

Example 3.5. Let $U = \{l, m, n\}$. We define IVFSs, P' and Q' on U and $U \times U$, respectively, as defined in Tables 3 & 4. The graphical representation of The IVFG $R' = (P', Q')$ is shown in Figure 2.

Table 3: An IVFS $\mathbb{P}'$ on X

U	***L***	***M***	***N***
$\mathbb{P}'$	[0.2,0.5]	[0.6,0.8]	[0.5,0.9]

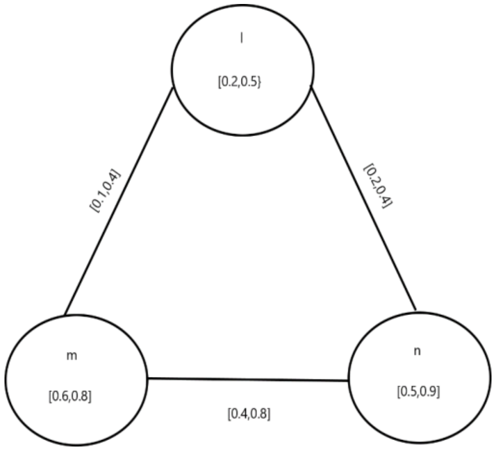
Source: Author's own

Table 4: An IVFS set $\mathbb{Q}'$ on $U \times U$

N' ⊆ U × U	***ln***	***Mn***	***Lm***
$\mathbb{Q}'$	[0.2,0.4]	[0.4,0.8]	[0.1,0.4]

Source: Author's own

Figure 2: An IVFG on *U*



Source: Author's own

Definition 3.6. A cubic set (CS) C on X is prescribed by mappings $\Psi_{\mathrm{C}} = [\Psi^{-}, \Psi^{+}]: X \rightarrow D\,[0, 1]$, and $\Psi'_{\mathrm{C}}: X \rightarrow [0, 1]$,

where Ψ_C and Ψ'_{C} is an IVFS and FS on X, respectively. A CS can be represented as

C = $\{(u_s, [\Psi_C^{-}(u_s), \Psi_C^{+}(u_s)], \Psi_{\mathrm{C}}{'}(u_s)): u_s \in X\}$,

where $[\Psi_C(\upsilon_s), \Psi_C(\upsilon_s)]$ and Ψ_C are the interval-valued fuzzy membership value and fuzzy membership value at υ_s, respectively.

The support and support length of C is defined as supp(C) = $\{u_s \in X \mid \Psi_{\mathrm{C}}^{-}(u_s) \neq 0, \Psi'_{\mathrm{C}} \neq 0\}$ and s(C) = |supp(C)|, respectively. The core and core length of C is core(C) = $\{u_s \in X \mid \Psi_{\mathrm{C}}^{-}(u_s) = 1, \Psi'_{\mathrm{C}} = 1\}$ and c(C) = |core(C)|, respectively. The height of cubic set C is $h(C) = \left([h^{-}(C), h^{+}(C)], h'(C)\right) =$ $([\max\psi_{C^{-}}(u_s), \max\psi_{C^{+}}(u_s)], \max\psi_{C'}(u_s).)$ The CS is called normal if $h(C) = 1$.

$$P^{*} = \{\left([\Psi_{P^{*}}^{-}(\upsilon_s), \Psi_{P^{*}}^{+}(\upsilon_s)], \Psi'_{P^{*}(\upsilon_s)}\right\},$$

$$Q^* = \{[\psi^-_{Q^*}(v_{s-1}, v_s), \psi^+_{Q^*}(v_{s-1}, v_s)], \psi'_{Q^*}(v_{s-1}, v_s)\},$$

are CSs on U and $U \times U$, so that

$$\psi^-_{Q^*}(v_{s-1}, v_s) \leq \min\{\psi^-_{P^*}(v_{s-1}), \psi^-_{P^*}(v_s)\}$$

$$\psi^+_{Q^*}(v_{s-1}, v_s) \leq \min\{\psi^+_{P^*}(v_{s-1}), \psi^+_{P^*}(v_s)\},$$

$$\psi'_{Q^*}(v_{s-1}, v_s) \leq \min\{\psi'_{P^*}(v_{s-1}), \psi'_{P^*}(v_s)\}.$$

Example 3.7: Suppose $U = \{l, m, n\}$. The membership values of CS P^* on U and Q^* on $U \times U$ are given in Tables 5 & 6. The CFG corresponding to the above data is shown in Figure 3.

Table 5: A CS on U

U	***L***	***M***	***N***
*P**	([0.2,0.6],0.5)	([0.3,0.9],0.2)	([0.4,0.85],0.64)

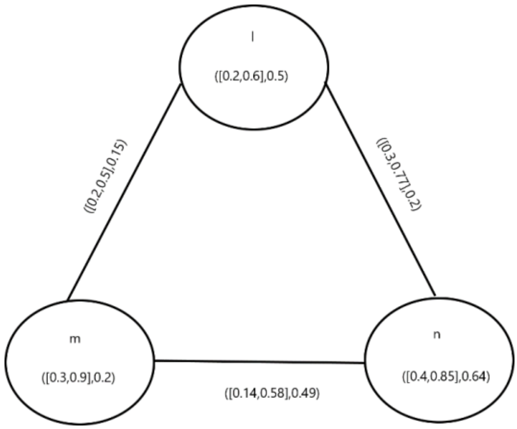
Source: Author's own

Table 6: A CS on U × U

M* ⊆ U × U	***lm***	***Ln***	***mn***
*Q**	([0.2,0.5],0.15)	([0.3,0.77],0.2)	([0.14,0.58],0.49)

Source: Author's own

Figure 3: A CFG on *U*



Source: Author's own

Definition 3.8. (Atanassov, 1999) Let X be a non-empty set. An IFS I over X is defined as

$I = (\upsilon_s, \check{\mu}_I(\upsilon_s), \check{\nu}_I(\upsilon_s))$,

where $\check{\mu}_I: X \to [0, 1]$, and $\check{\nu}_I: X \to [0, 1]$, denote the membership and non-membership function, respectively, such that for all $\upsilon_s \in X$, $0 \leq \check{\mu}_I(\upsilon_s) + \check{\nu}_I(\upsilon_s) \leq 1$.

Definition 3.9. (Muneeza and Abdullah, 2020) An ICFS I_C over a non-empty set X is defined as:

$I_C = \{ (\upsilon_s, ([\check{\mu}_I^-, \check{\mu}_I^+], \check{\mu}_I), ([\check{\nu}_I^-, \check{\nu}_I^+], \check{\nu}_I)) : \upsilon_s \in X\}$,

where $([\check{\mu}_I^-, \check{\mu}_I^+], \check{\mu}_I)$ and $([\check{\nu}_I^-, \check{\nu}_I^+], \check{\nu}_I)$ are the cubic numbers and denotes the membership and non-membership grades of I_C.

Definition 3.10. (Pramanik, et al., 2016) Let X be a non-empty set, an ICFG $\tilde{R}$ is a pair $(\tilde{P}, \tilde{Q})$, where

$$\widetilde{P} = \{(\upsilon_s, \left([\breve{\mu}_P^-(\upsilon_s), \breve{\mu}_P^+(\upsilon_s)], \breve{\mu}_P(\upsilon_s)\right), \left([\breve{\nu}_P^-(\upsilon_s), \nu_P^+(\upsilon_s)], \breve{\nu}_P(\upsilon_s)\right))\}$$

is an ICFS on X and

$$\widetilde{Q} = \left\{\left(\begin{array}{c}(\upsilon_{s-1}, \upsilon_s), \left(\begin{array}{c}\left[\breve{\mu}_Q^-((\upsilon_{s-1}, \upsilon_s), \breve{\mu}_Q^+((\upsilon_{s-1}, \upsilon_s)\right], \\ \breve{\mu}_Q((\upsilon_{s-1}, \upsilon_s)\end{array}\right), \\ \left(\begin{array}{c}\left[\breve{\nu}_Q^-((\upsilon_{s-1}, \upsilon_s), \nu_Q^+((\upsilon_{s-1}, \upsilon_s)\right], \\ \breve{\nu}_P((\upsilon_{s-1}, \upsilon_s)\end{array}\right)\end{array}\right)\right\}$$

is an ICFS on $X \times X$ such that for every $\upsilon_{s-1}, \upsilon_s \in X$,

$\breve{\mu}_Q^-((\upsilon_{s-1}, \upsilon_s) \leq \min\{\breve{\mu}_P^-(\upsilon_{s-1}), \breve{\mu}_P^-(\upsilon_s)\}$,

$\breve{\mu}_Q^+((\upsilon_{s-1}, \upsilon_s) \leq \min\{\breve{\mu}_P^+(\upsilon_{s-1}), \breve{\mu}_P^+(\upsilon_s)\}$,

$\breve{\mu}_Q((\upsilon_{s-1}, \upsilon_s) \leq \min\{\breve{\mu}_P(\upsilon_{s-1}), \breve{\mu}_P(\upsilon_s)\}$,

$\breve{\nu}_Q^-((\upsilon_{s-1}, \upsilon_s) \geq \max\{\breve{\nu}_P^-(\upsilon_{s-1}), \breve{\nu}_P^-(\upsilon_s)\}$,

$\nu_Q^+((\upsilon_{s-1}, \upsilon_s) \geq \max\{\nu_P^+(\upsilon_{s-1}), \nu_P^+(\upsilon_s)\}$,

$\breve{\nu}_P((\upsilon_{s-1}, \upsilon_s) \geq \max\{\breve{\nu}_P(\upsilon_{s-1}), \breve{\nu}_P(\upsilon_s)\}$.

Definition 3.11. (Muneeza and Abdullah, 2020) Let I_C be an ICFN defined as $(([\check{\mu}_I^-, \check{\mu}_I^+], \check{\mu}_I), ([\check{\nu}_I^-, \check{\nu}_I^+], \check{\nu}_I))$. Then, score function of I_C is defined as:

$$S(I_c) = \frac{\breve{\mu}_I^- + \breve{\mu}_I^+ + \breve{\mu}_I - \breve{\nu}_I^- + \breve{\nu}_I^+ - \breve{\nu}_I}{3},$$

such that $-1 \leq S(I_c) \leq 1$.

The accuracy function I_c is given as:

$$H(I_c) = \frac{\check{\mu}_I^- + \check{\mu}_I^+ + \check{\mu}_I + \check{\nu}_I^- + \check{\nu}_I^+ + \check{\nu}_I}{3},$$

such that $-1 \leq H(I_c) \leq 1$.

4. ARAS TECHNIQUE ON ICFG

The algorithm used by the ICF-ARAS method, facilitating the solution of the MCDM issue in ICF, is developed in this phase. It requires that experts give significant weights of criteria in order to establish the methodology. Also, used in this method is the ICF choice matrix, which is normalized with respect to the type of criterion. Then, the optimal function is determined in this method by adding the ideal solution and finding the weighted matrix of norms. Lastly, the utility level is determined depending on the accuracy measure of each option. Finally, the options are prioritized based on their utility.

Step 1: Constructing ICF decision matrix

For the alternatives $\mathbb{P}_1, \mathbb{P}_2. \dots, \mathbb{P}_m$ and the criteria $C_1, C_2 \dots, C_n$, i.e., benefit $\mathbb{Q}_t^b$ and cost (non-benefit) criteria $\mathbb{Q}_t^{nb}$ the ICFDM is represented as

$$M = \left[< \left(\left[\mu_{ij}^-, \mu_{ij}^+\right], \mu_{ij}\right), \left(\left[\nu_{ij}^-, \nu_{ij}^+\right], \nu_{ij}\right) >\right]_{m \times n},$$

where $\left(\left[\mu_{ij}^-, \mu_{ij}^+\right], \mu_{ij}\right)$ and $\left(\left[\nu_{ij}^-, \nu_{ij}^+\right], \nu_{ij}\right)$ represents the membership and non-membership grades of the edge between ith alternatives and jth criteria, respectively. It is given in eq. (1)

$$M = \begin{bmatrix} (([\check{\mu}_{11}^-, \check{\mu}_{11}^+], \check{\mu}_{11}), ([\check{\nu}_{11}^-, \check{\nu}_{11}^+], \check{\nu}_{11})) & (([\check{\mu}_{12}^-, \check{\mu}_{12}^+], \check{\mu}_{12}), ([\check{\nu}_{12}^-, \check{\nu}_{12}^+], \check{\nu}_{12})) & \cdots & (([\check{\mu}_{1n}^-, \check{\mu}_{1n}^+], \check{\mu}_{1n}), ([\check{\nu}_{1n}^-, \check{\nu}_{1n}^+], \check{\nu}_{11})) \\ (([\check{\mu}_{21}^-, \check{\mu}_{21}^+], \check{\mu}_{21}), ([\check{\nu}_{21}^-, \check{\nu}_{21}^+], \check{\nu}_{21})) & (([\check{\mu}_{22}^-, \check{\mu}_{22}^+], \check{\mu}_{22}), ([\check{\nu}_{22}^-, \check{\nu}_{22}^+], \check{\nu}_{22})) & \cdots & (([\check{\mu}_{2n}^-, \check{\mu}_{2n}^+], \check{\mu}_{2n}), ([\check{\nu}_{2n}^-, \check{\nu}_{2n}^+], \check{\nu}_{2n})) \\ \vdots & \vdots & \ddots & \vdots \\ (([\check{\mu}_{m1}^-, \check{\mu}_{m1}^+], \check{\mu}_{m1}), ([\check{\nu}_{m1}^-, \check{\nu}_{m1}^+], \check{\nu}_{m1})) & (([\check{\mu}_{m2}^-, \check{\mu}_{m2}^+], \check{\mu}_{m2}), ([\check{\nu}_{m2}^-, \check{\nu}_{m2}^+], \check{\nu}_{m2})) & \vdots & (([\check{\mu}_{mn}^-, \check{\mu}_{mn}^+], \check{\mu}_{mn}), ([\check{\nu}_{mn}^-, \check{\nu}_{mn}^+], \check{\nu}_{mn})) \end{bmatrix} \quad (1)$$

where $i = 1, 2 \dots, m$ and $j = 1, 2 \dots, n$.

Step 2: Rescaling the ICFDM

The normalization of ICFDM is given below in eq. (2):

$\widetilde{M} =$

$$\begin{bmatrix} \left(\left([\widetilde{\check{\mu}_{11}^{-}},\widetilde{\check{\mu}_{11}^{+}}],\widetilde{\check{\mu}_{11}}\right),\left([\widetilde{\check{\nu}_{11}^{-}},\widetilde{\check{\nu}_{11}^{+}}],\widetilde{\check{\nu}_{11}}\right)\right) & \left(\left([\widetilde{\check{\mu}_{12}^{-}},\widetilde{\check{\mu}_{12}^{+}}],\widetilde{\check{\mu}_{12}}\right),\left([\widetilde{\check{\nu}_{12}^{-}},\widetilde{\check{\nu}_{12}^{+}}],\widetilde{\check{\nu}_{12}}\right)\right) & \cdots & \left(\left([\widetilde{\check{\mu}_{1n}^{-}},\widetilde{\check{\mu}_{1n}^{+}}],\widetilde{\check{\mu}_{1n}}\right),\left([\widetilde{\check{\nu}_{1n}^{-}},\widetilde{\check{\nu}_{1n}^{+}}],\widetilde{\check{\nu}_{1n}}\right)\right) \\ \left(\left([\widetilde{\check{\mu}_{21}^{-}},\widetilde{\check{\mu}_{21}^{+}}],\widetilde{\check{\mu}_{21}}\right),\left([\widetilde{\check{\nu}_{21}^{-}},\widetilde{\check{\nu}_{21}^{+}}],\widetilde{\check{\nu}_{21}}\right)\right) & \left(\left([\widetilde{\check{\mu}_{22}^{-}},\widetilde{\check{\mu}_{22}^{+}}],\widetilde{\check{\mu}_{22}}\right),\left([\widetilde{\check{\nu}_{22}^{-}},\widetilde{\check{\nu}_{22}^{+}}],\widetilde{\check{\nu}_{22}}\right)\right) & \cdots & \left(\left([\widetilde{\check{\mu}_{2n}^{-}},\widetilde{\check{\mu}_{2n}^{+}}],\widetilde{\check{\mu}_{2n}}\right),\left([\widetilde{\check{\nu}_{2n}^{-}},\widetilde{\check{\nu}_{2n}^{+}}],\widetilde{\check{\nu}_{2n}}\right)\right) \\ \vdots & \vdots & \ddots & \vdots \\ \left(\left([\widetilde{\check{\mu}_{m1}^{-}},\widetilde{\check{\mu}_{m1}^{+}}],\widetilde{\check{\mu}_{m1}}\right),\left([\widetilde{\check{\nu}_{m1}^{-}},\widetilde{\check{\nu}_{m1}^{+}}],\widetilde{\check{\nu}_{m1}}\right)\right) & \left(\left([\widetilde{\check{\mu}_{m2}^{-}},\widetilde{\check{\mu}_{m2}^{+}}],\widetilde{\check{\mu}_{m2}}\right),\left([\widetilde{\check{\nu}_{m2}^{-}},\widetilde{\check{\nu}_{m1}^{+}}],\widetilde{\check{\nu}_{m1}}\right)\right) & \cdots & \left(\left([\widetilde{\check{\mu}_{mn}^{-}},\widetilde{\check{\mu}_{mn}^{+}}],\widetilde{\check{\mu}_{mn}}\right),\left([\widetilde{\check{\nu}_{mn}^{-}},\widetilde{\check{\nu}_{mn}^{+}}],\widetilde{\check{\nu}_{mn}}\right)\right) \end{bmatrix} \tag{2}$$

where for beneficial criteria $\mathbb{Q}_t^b$, the values are given in eq. (3):

$$\widetilde{\check{\mu}_{ij}^{-}} = \frac{\check{\mu}_{ij}^{-}}{\tilde{s}}, \widetilde{\check{\mu}_{ij}^{+}} = \frac{\check{\mu}_{ij}^{+}}{\tilde{s}}, \widetilde{\check{\mu}_{ij}} = \frac{\check{\mu}_{ij}}{\tilde{s}}, \ \widetilde{\check{\nu}_{ij}^{-}} = \frac{\check{\nu}_{ij}^{-}}{\tilde{s}}, \widetilde{\check{\nu}_{ij}^{+}} = \frac{\check{\nu}_{ij}^{+}}{\tilde{s}}, \widetilde{\check{\nu}_{ij}} = \frac{\check{\nu}_{ij}}{\tilde{s}}, \tag{3}$$

Where,

$$\tilde{s} = \sum_{i=1}^{m}\left(\check{\mu}_{ij}^{-} + \check{\mu}_{ij}^{+} + \check{\mu}_{ij} + \check{\nu}_{ij}^{-} + \check{\nu}_{ij}^{+} + \check{\nu}_{ij}\right)$$

For cost criteria ($\mathbb{Q}_t^{nb}$), the values are given in eq. (4)

$$\widetilde{\check{\mu}_{ij}^{-}} = \frac{1/\check{\mu}_{ij}^{-}}{\tilde{s}\prime}, \widetilde{\check{\mu}_{ij}^{+}} = \frac{1/\check{\mu}_{ij}^{+}}{\tilde{s}\prime}, \widetilde{\check{\mu}_{ij}} = \frac{1/\check{\mu}_{ij}}{\tilde{s}\prime}, \ \widetilde{\check{\nu}_{ij}^{-}} = \frac{1/\check{\nu}_{ij}^{-}}{\tilde{s}\prime}, \widetilde{\check{\nu}_{ij}^{+}} = \frac{1/\check{\nu}_{ij}^{+}}{\tilde{s}\prime}, \widetilde{\check{\nu}_{ij}} = \frac{1/\check{\nu}_{ij}}{\tilde{s}\prime} \tag{4}$$

Where,

$$\tilde{s}' = \frac{1}{\sum_{i=1}^{m}\left(\check{\mu}_{ij}^{-}+\check{\mu}_{ij}^{+}+\check{\mu}_{ij}+\check{\nu}_{ij}^{-}+\check{\nu}_{ij}^{+}+\check{\nu}_{ij}\right)}$$

Step 3: Adding best alternative

The best alternative is distinguished by considering the largest and least value of the ICFN for each column. The (m + 1 × n) matrix is given in eq. (5)

$$\widetilde{M} = \begin{bmatrix} \left(\left([\widetilde{\check{\mu}_{01}^{-}},\widetilde{\check{\mu}_{01}^{+}}],\widetilde{\check{\mu}_{11}}\right),\left([\widetilde{\check{\nu}_{01}^{-}},\widetilde{\check{\nu}_{01}^{+}}],\widetilde{\check{\nu}_{01}}\right)\right) & \left(\left([\widetilde{\check{\mu}_{02}^{-}},\widetilde{\check{\mu}_{02}^{+}}],\widetilde{\check{\mu}_{02}}\right),\left([\widetilde{\check{\nu}_{02}^{-}},\widetilde{\check{\nu}_{02}^{+}}],\widetilde{\check{\nu}_{02}}\right)\right) & \cdots & \left(\left([\widetilde{\check{\mu}_{0n}^{-}},\widetilde{\check{\mu}_{0n}^{+}}],\widetilde{\check{\mu}_{0n}}\right),\left([\widetilde{\check{\nu}_{0n}^{-}},\widetilde{\check{\nu}_{0n}^{+}}],\widetilde{\check{\nu}_{0n}}\right)\right) \\ \left(\left([\widetilde{\check{\mu}_{11}^{-}},\widetilde{\check{\mu}_{11}^{+}}],\widetilde{\check{\mu}_{11}}\right),\left([\widetilde{\check{\nu}_{11}^{-}},\widetilde{\check{\nu}_{11}^{+}}],\widetilde{\check{\nu}_{11}}\right)\right) & \left(\left([\widetilde{\check{\mu}_{12}^{-}},\widetilde{\check{\mu}_{12}^{+}}],\widetilde{\check{\mu}_{12}}\right),\left([\widetilde{\check{\nu}_{12}^{-}},\widetilde{\check{\nu}_{12}^{+}}],\widetilde{\check{\nu}_{12}}\right)\right) & \cdots & \left(\left([\widetilde{\check{\mu}_{1n}^{-}},\widetilde{\check{\mu}_{1n}^{+}}],\widetilde{\check{\mu}_{1n}}\right),\left([\widetilde{\check{\nu}_{1n}^{-}},\widetilde{\check{\nu}_{1n}^{+}}],\widetilde{\check{\nu}_{1n}}\right)\right) \\ \vdots & \vdots & \ddots & \vdots \\ \left(\left([\widetilde{\check{\mu}_{m1}^{-}},\widetilde{\check{\mu}_{m1}^{+}}],\widetilde{\check{\mu}_{m1}}\right),\left([\widetilde{\check{\nu}_{m1}^{-}},\widetilde{\check{\nu}_{m1}^{+}}],\widetilde{\check{\nu}_{m1}}\right)\right) & \left(\left([\widetilde{\check{\mu}_{m2}^{-}},\widetilde{\check{\mu}_{m2}^{+}}],\widetilde{\check{\mu}_{m2}}\right),\left([\widetilde{\check{\nu}_{m2}^{-}},\widetilde{\check{\nu}_{m1}^{+}}],\widetilde{\check{\nu}_{m1}}\right)\right) & \cdots & \left(\left([\widetilde{\check{\mu}_{mn}^{-}},\widetilde{\check{\mu}_{mn}^{+}}],\widetilde{\check{\mu}_{mn}}\right),\left([\widetilde{\check{\nu}_{mn}^{-}},\widetilde{\check{\nu}_{mn}^{+}}],\widetilde{\check{\nu}_{mn}}\right)\right) \end{bmatrix} \tag{5}$$

For Benefit criteria ($\mathbb{Q}_t^b$), we have eq. (6)

$$\left(\left(\left([\widetilde{\check{\mu}_{01}^{-}},\widetilde{\check{\mu}_{01}^{+}}],\widetilde{\check{\mu}_{11}}\right),\left([\widetilde{\check{\nu}_{01}^{-}},\widetilde{\check{\nu}_{01}^{+}}],\widetilde{\check{\nu}_{01}}\right)\right)\right) = \left[\left(\max_{i=1}\left([\widetilde{\check{\mu}_{ij}^{-}},\widetilde{\check{\mu}_{ij}^{+}}],\widetilde{\check{\mu}_{ij}}\right)\right),\left(\min_{i=1}\left([\widetilde{\check{\nu}_{ij}^{-}},\widetilde{\check{\nu}_{ij}^{+}}],\widetilde{\check{\nu}_{ij}}\right)\right)\right] \tag{6}$$

For Cost criteria ($\mathbb{Q}_t^b$), we have eq. (7)

$$\left(\left(([\widetilde{\check{\mu}_{01}^{-}},\widetilde{\check{\mu}_{01}^{+}}],\widetilde{\check{\mu}_{11}}),([\widetilde{\check{v}_{01}^{-}},\widetilde{\check{v}_{01}^{+}}],\widetilde{\check{v}_{01}})\right)\right) = \left[\left(\min_{i=1}([\widetilde{\check{\mu}_{ij}^{-}},\widetilde{\check{\mu}_{ij}^{+}}],\widetilde{\check{\mu}_{ij}})\right),(\max_{i=1}([\widetilde{\check{v}_{ij}^{-}},\widetilde{\check{v}_{ij}^{+}}],\widetilde{\check{v}_{ij}}))\right] \tag{7}$$

$j = 1, 2 \ldots, n.$

Step 4: Determining the weighted normalized ICFDM

The entropy weights suggested by the experts in eq. (8)

$$\widetilde{\omega} = (\widetilde{\omega}_1, \widetilde{\omega}_2, \ldots, \widetilde{\omega}_n) \tag{8}$$

We get the weighted normalized ICFDM given in eq, (9) by multiplying $\widetilde{\omega}$ with $\widetilde{M}$ in eq. (5).

$$\widetilde{W} = \begin{bmatrix} \left(([\widetilde{\check{\alpha}_{01}^{-}},\widetilde{\check{\alpha}_{01}^{+}}],\widetilde{\check{\alpha}_{11}}),([\widetilde{\check{\beta}_{01}^{-}},\widetilde{\check{\beta}_{01}^{+}}],\widetilde{\check{\beta}_{01}})\right) & \left(([\widetilde{\check{\alpha}_{02}^{-}},\widetilde{\check{\alpha}_{02}^{+}}],\widetilde{\check{\alpha}_{02}}),([\widetilde{\check{\beta}_{02}^{-}},\widetilde{\check{\beta}_{02}^{+}}],\widetilde{\check{\beta}_{02}})\right) & \cdots & \left(([\widetilde{\check{\alpha}_{0n}^{-}},\widetilde{\check{\alpha}_{0n}^{+}}],\widetilde{\check{\alpha}_{0n}}),([\widetilde{\check{\beta}_{0n}^{-}},\widetilde{\check{\beta}_{0n}^{+}}],\widetilde{\check{\beta}_{0n}})\right) \\ \left(([\widetilde{\check{\alpha}_{11}^{-}},\widetilde{\check{\alpha}_{11}^{+}}],\widetilde{\check{\alpha}_{11}}),([\widetilde{\check{\beta}_{11}^{-}},\widetilde{\check{\beta}_{11}^{+}}],\widetilde{\check{\beta}_{11}})\right) & \left(([\widetilde{\check{\alpha}_{12}^{-}},\widetilde{\check{\alpha}_{12}^{+}}],\widetilde{\check{\alpha}_{12}}),([\widetilde{\check{\beta}_{12}^{-}},\widetilde{\check{\beta}_{12}^{+}}],\widetilde{\check{\beta}_{12}})\right) & \cdots & \left(([\widetilde{\check{\alpha}_{1n}^{-}},\widetilde{\check{\alpha}_{1n}^{+}}],\widetilde{\check{\alpha}_{1n}}),([\widetilde{\check{\beta}_{1n}^{-}},\widetilde{\check{\beta}_{1n}^{+}}],\widetilde{\check{\beta}_{1n}})\right) \\ \vdots & \vdots & \ddots & \vdots \\ \left(([\widetilde{\check{\alpha}_{m1}^{-}},\widetilde{\check{\alpha}_{m1}^{+}}],\widetilde{\alpha_{m1}}),([\widetilde{\check{\beta}_{m1}^{-}},\widetilde{\check{\beta}_{m1}^{+}}],\widetilde{\check{\beta}_{m1}})\right) & \left(([\widetilde{\check{\alpha}_{m2}^{-}},\widetilde{\check{\alpha}_{m2}^{+}}],\widetilde{\check{\alpha}_{m2}}),([\widetilde{\check{\beta}_{m2}^{-}},\widetilde{\check{\beta}_{m1}^{+}}],\widetilde{\check{\beta}_{m1}})\right) & \cdots & \left(([\widetilde{\check{\alpha}_{mn}^{-}},\widetilde{\check{\alpha}_{mn}^{+}}],\widetilde{\check{\alpha}_{mn}}),([\widetilde{\check{\beta}_{mn}^{-}},\widetilde{\check{\beta}_{mn}^{+}}],\widetilde{\check{\beta}_{mn}})\right) \end{bmatrix}, \tag{9}$$

such that

$$\left(([\widetilde{\check{\alpha}_{ij}^{-}},\widetilde{\check{\alpha}_{ij}^{+}}],\widetilde{\check{\alpha}_{ij}}),([\widetilde{\check{\beta}_{ij}^{-}},\widetilde{\check{\beta}_{ij}^{+}}],\widetilde{\check{\beta}_{ij}})\right) = \omega_j \left[\left(([\widetilde{\check{\mu}_{ij}^{-}},\widetilde{\check{\mu}_{ij}^{+}}],\widetilde{\check{\mu}_{ij}}),([\widetilde{\check{v}_{ij}^{-}},\widetilde{\check{v}_{ij}^{+}}],\widetilde{\check{v}_{ij}})\right)\right]$$

Step 5 Optimality function

For each row for i=1, 2,,m, it is computed as:

$$\widetilde{O_P} = \left(([\widetilde{O_{pi}^{-}},\widetilde{O_{pi}^{+}}],\widetilde{O_{pi}}),([\widetilde{Q_{pi}^{-}},\widetilde{Q_{pi}^{+}}],\widetilde{Q_{pi}})\right),$$

$$\widetilde{O_{pi}^{-}} = \sum_{j=1}^{n} \widetilde{\check{\alpha}_{ij}^{-}},\ \widetilde{O_{pi}^{+}} = \sum_{j=1}^{n} \widetilde{\check{\alpha}_{ij}^{+}},\ \widetilde{O_{pi}} = \sum_{j=1}^{n} \widetilde{\check{\alpha}_{ij}}$$

$$\widetilde{Q_{pi}^{-}} = \sum_{i=1}^{n} \widetilde{\check{\beta}_{ij}^{-}},\ \widetilde{Q_{pi}^{+}} = \sum_{i=1}^{n} \widetilde{\check{\beta}_{ij}^{+}},\ \widetilde{Q_{pi}} = \sum_{i=1}^{n} \widetilde{\check{\beta}_{ij}}$$

The accuracy value for each alternative $\mathbb{P}_i$ for i=0,1, 2,…, is computed as:

$$\widetilde{\mathbb{P}_i} = \frac{\widetilde{O_{pi}^{-}}+\widetilde{O_{pi}^{+}}+\widetilde{O_{pi}}-\widetilde{Q_{pi}^{-}}+\widetilde{Q_{pi}^{+}}-\widetilde{Q_{pi}}}{3}.$$

Step 6: Determining the utility degree

For each alternative, it is calculated as $\widetilde{U_i} = \frac{\widetilde{\mathbb{P}_i}}{\widetilde{\mathbb{P}_0}}$, so that $\mathbb{P}_0 = max\ \mathbb{P}_i$.

Step 7: Ranking the alternatives

The alternatives are ranked by calculating and comparing each alternative's utility degree. the alternatives arranged in descending order. The best option is the one with the highest utility degree. The flowchart of the proposed technique is given in Figure 4. The expert judgement, based on institutional priorities and academic standards, provides the criterion weight in the proposed framework for making academic placement decisions. The current study assumes that all DM's reach a consensus on the selection process, however, as illustrated by the present study, the framework can naturally be extended to incorporate group decision-making mechanisms, such as averaged or aggregated expert opinion. The flexibility of the framework also allows for alignment with institutional recruitment policies and collective governance practices typically embraced by many higher education institutions.

5. DECISION-MAKING FRAMEWORK FOR TEACHER SELECTION

Selection of teachers for an educational institution is a critical process as it entails the consideration of a number of aspects such as qualifications and teaching experience, communication and research abilities, and student and class management. It is a major factor in determining the quality of teaching that is required by the students. Many a times, it entails a certain amount of subjective judgment and risk. It is a fact that institutions follow a tough procedure in order to ensure that those selected as teachers possess qualifications in academics as well as an inclination towards teaching and the ability to hold their audience in thrall. There are a number of steps involved in teacher selection. Four candidates: $\mathbb{P}_1$, $\mathbb{P}_2$, $\mathbb{P}_3$, and $\mathbb{P}_4$ advance to a more detailed evaluation. To identify the most suitable candidate, the decision-maker evaluates them based on four key criteria: C_1: Qualification,

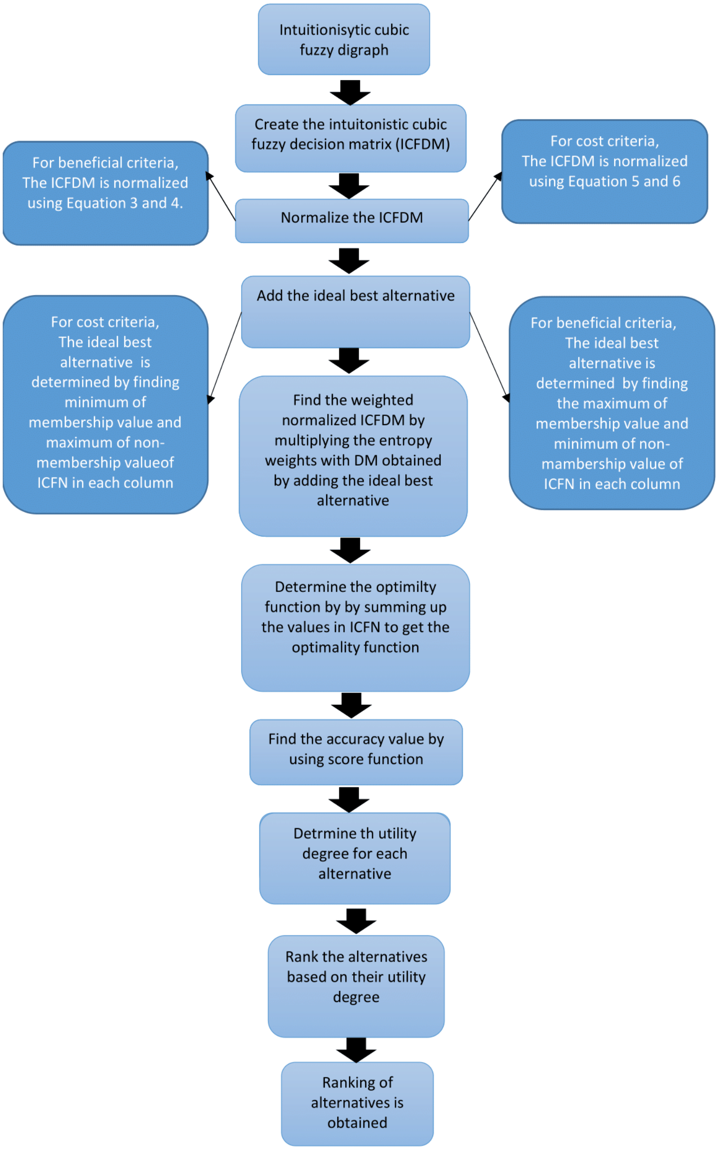
C_2: Interview results,

C_3: Teaching experience,

C_4: Communication skills

It is important to note that faculty recruitment inherently involves human judgment, which may be influenced by cognitive biases, institutional preferences, and varying interpretations of evaluation criteria. The intuitionistic cubic fuzzy representation allows decision-makers to express hesitation and partial belief, thereby reducing the rigidity of crisp scoring systems and mitigating the impact of individual bias. By accommodating both membership and non-membership degrees, the proposed framework provides a more realistic representation of expert assessments

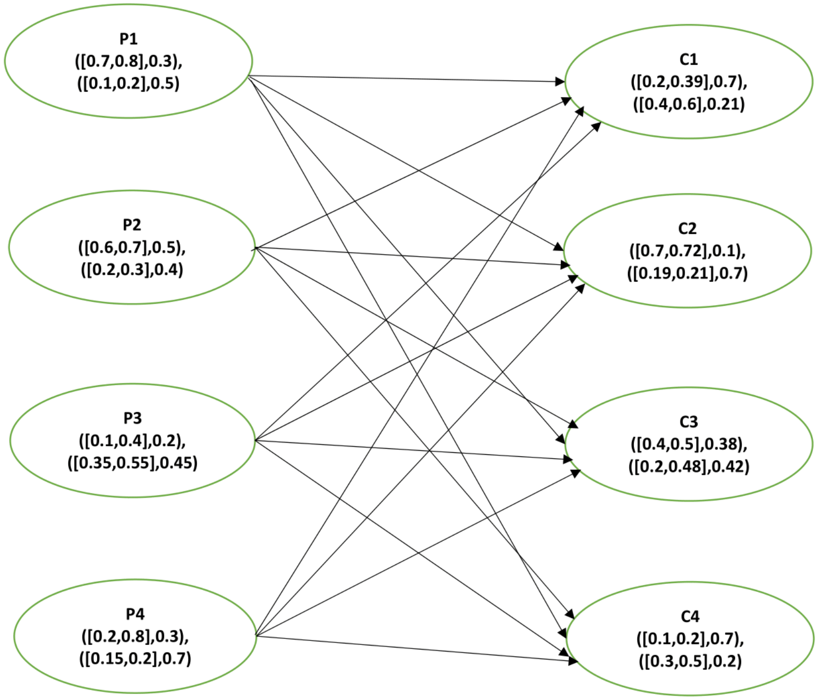
Figure 4: Flow Chart of ICFG based ARAS Technique



Source: Author's own

Suppose the decision-maker expresses his or her preference regarding candidates for criteria by using ICFN as given in Figure 5.

Figure 5: Intuitionistic Cubic Fuzzy Decision Graph of Alternatives and Criteria



Source: Author's own

The intuitionistic cubic fuzzy number is significant since it handles uncertainties more precisely and in intuitionistic numbers, which are cubic numbers including the membership and non-membership values. The following points can be taken under consideration to model this problem:

- The vertices of the graph are defined by the four candidates and the four criteria.
- The vertices (candidates) possess a membership value in the form of an ICFS, representing the current and previous minimum and maximum membership and non-membership value of the chances of hiring the teacher.

- The vertices used as the selection criteria do have a value of membership. The membership value is in ICFS form and expresses the requirements of the institute for the particular criteria.
- There is an edge between the vertices if the candidate can meet the requirements. The edge membership value represents the degree to which the candidate meets the requirements.

The ARAS technique is then applied to the ICFDG given in Figure 5. The best optimal candidate for the teaching position is then selected. The breakdown of the computation is illustrated as below:

Step 1: The following Table 7 describe the values given by the decision-makers.

Table 7: Intuitionistic Cubic Fuzzy Decision Matrix of Alternative and Criteria

Alternatives	C_1	C_2	C_3	C_4
$\mathbb{P}_1$	$\begin{pmatrix}([0.2,0.31],0.25),\\([0.05,0.2],0.2)\end{pmatrix}$	$\begin{pmatrix}([0.6,0.7],0.1),\\([0.1,0.2],0.4)\end{pmatrix}$	$\begin{pmatrix}([0.3,0.45],0.3),\\([0.1,0.19],0.4)\end{pmatrix}$	$\begin{pmatrix}([0.1,0.2],0.2),\\([0.1,0.15],0.15)\end{pmatrix}$
$\mathbb{P}_2$	$\begin{pmatrix}([0.2,0.35],0.49),\\([0.2,0.25],0.2)\end{pmatrix}$	$\begin{pmatrix}([0.6,0.7],0.1),\\([0.15,0.2],0.3)\end{pmatrix}$	$\begin{pmatrix}([0.4,0.5],0.35),\\([0.2,0.29],0.39)\end{pmatrix}$	$\begin{pmatrix}([0.1,0.15],0.4),\\([0.1,0.3],0.2)\end{pmatrix}$
$\mathbb{P}_3$	$\begin{pmatrix}([0.1,0.3],0.1),\\([0.3,0.55],0.2)\end{pmatrix}$	$\begin{pmatrix}([0.1,0.2],0.1),\\([0.17,0.2],0.4)\end{pmatrix}$	$\begin{pmatrix}([0.1,0.4],0.2),\\([0.2,0.4],0.41)\end{pmatrix}$	$\begin{pmatrix}([0.1,0.15],0.2),\\([0.2,0.24],0.2)\end{pmatrix}$
$\mathbb{P}_4$	$\begin{pmatrix}([0.2,0.21],0.21),\\([0.15,0.2],0.18)\end{pmatrix}$	$\begin{pmatrix}([0.2,0.7],0.1),\\([0.1,0.2],0.2)\end{pmatrix}$	$\begin{pmatrix}([0.15,0.45],0.2),\\([0.1,0.22],0.4)\end{pmatrix}$	$\begin{pmatrix}([0.05,0.15],0.3),\\([0.1,0.2],0.15)\end{pmatrix}$

Source: Author's own

Step 2: The normalized ICF decision matrix is given in Table 8:

Table 8: Normalized Decision Matrix of Alternatives and Criteria

Alternatives	C_1	C_2	C_3	C_4
$\mathbb{P}_1$	$\begin{pmatrix}\begin{pmatrix}[0.0357,0.0554],\\0.0446\end{pmatrix}\\\begin{pmatrix}[0.0089,0.0375],\\0.0357\end{pmatrix}\end{pmatrix}$	$\begin{pmatrix}\begin{pmatrix}[0.0088,0.1026],\\0.0147\end{pmatrix}\\\begin{pmatrix}[0.147,0.0293],\\0.0587\end{pmatrix}\end{pmatrix}$	$\begin{pmatrix}\begin{pmatrix}[0.0423,0.0634],\\0.0423\end{pmatrix}\\\begin{pmatrix}[0.0141,0.0268],\\0.563\end{pmatrix}\end{pmatrix}$	$\begin{pmatrix}\begin{pmatrix}[0.0239,0.0477],\\0.0477\end{pmatrix}\\\begin{pmatrix}[0.0239,0.0358],\\0.0358\end{pmatrix}\end{pmatrix}$

$\mathbb{P}_2$	(([0.0357,0.0625], 0.0875) ([0.0357,0.0446], 0.0357))	(([0.0088,0.1026], 0.0147) ([0.0022,0.0293], 0.0044))	(([0.0563,0.0704], 0.0493) ([0.0282,0.0408], 0.0549))	(([0.0239,0.0358], 0.0955) ([0.0239,0.0716], 0.0477))
$\mathbb{P}_3$	(([0.0179,0.0536], 0.0179) ([0.0536,0.0982], 0.0357))	(([0.0147,0.0293], 0.0147) ([0.0249,0.0293], 0.0587))	(([0.0141,0.0563], 0.0282) ([0.0282,0.0573], 0.0577))	(([0.0239,0.0358], 0.0477) ([0.0477,0.0573], 0.0477))
$\mathbb{P}_4$	(([0.0357,0.0375], 0.0375) ([0.0268,0.0375], 0.0321))	(([0.0293,0.1026], 0.0147) ([0.0147,0.0293], 0.0293))	(([0.0211,0.0634], 0.0282) ([0.0141,0.031], 0.0563))	(([0.0119,0.0358], 0.0375) ([0.038,0.0477], 0.0358))

Source: Author's own

Step 3: After the addition of ideal best alternative, the normalized decision matrix is described in Table 9.

Table 9: Addition of Ideal Best Alternative in Normalized Decision Matrix

Alternatives	***C₁***	***C₂***	***C₃***	***C₄***
Ideal best	(([0.0357, 0.0625], $O.O$875,) ([0.0089,0.0357], 0.0321))	(([0.088,0,1026], 0.0147,) ([0.0147,0.0293], 0.0293))	(([0.0563,0.0704], 0.0493,) ([0.0141,0.0268], $O.O$549))	(([$O.O$239,0.0477,] 0.0955,) ([0.0239,0.0358], 0.0358))
$\mathbb{P}_1$	(([0.0357, 0.0554], 0.0446,) ([0.0089, 0.0357], 0.0357))	(([0.088, 0.1026], 0.0147,) ([0.0147, 0.0293], 0.0587))	(([0.0423, 0.0634], 0.0423,) ([0.0141, 0.0268], 0.0563))	(([0.0239,0.0477], 0.0955,) ([0.0239,0.0358], 0.0358))
$\mathbb{P}_2$	(([0.0357,0.0625], 0.0875,) ([0.0357,0.0446], 0.0357))	(([0.088, 0.1026], 0.0147,) ([0.022, 0.0293], 0.0044))	(([0.0563, 0.0704], 0.0493,) ([0.0282, 0.0408], 0.0549))	(([0.0239, 0.0358], 0.0955,) ([0.0239, 0.0716], 0.0477))
$\mathbb{P}_3$	(([0.0179,0.0539], 0.0179,) ([0.0536,0.0982], 0.0357))	(([0.0147, 0.0293], 0.0147,) ([0.0249, 0.0293], 0.0587))	(([0.0141, 0.0563], 0.0282,) ([0.0282, 0.0563], 0.0577))	(([0.0239, 0.0358], 0.0477,) ([0.0477, 0.0573], 0.0477))
$\mathbb{P}_4$	(([$O.O$357,0.0375], 0.0375) ([0.0268, $O.O$375], 0.0321))	(([0.0293, 0.1026], 0.0147,) ([0.0147, 0.0293], 0.0293))	(([0.0211, 0.0634], 0.0282,) ([0.0141, 0.031], 0.0563))	(([0.0119, 0.0358], 0.0716,) ([0.0238, 0.0477], 0.0358))

Source: Author's own

Step 4: The weights suggested by decision-makers assigned to each criterion are:

$$\widetilde{\omega} = (\widetilde{\omega}_1, \widetilde{\omega}_2, \widetilde{\omega}_3, \widetilde{\omega}_4) = (0.2, 0.35, 0.15, 0.3)$$

such that $\sum_{j=1}^{4} \widetilde{\omega}_j = 1$. The weighted normalized decision matrix is in Table 10

Alternatives	***C₁***	***C₂***	***C₃***	***C₄***
Ideal best	(([0.0071, 0.0125], 0.0175), ([0.0018, 0.0071], 0.0064))	(([0.0308, 0.0359], 0.0051), ([0.0051, 0.0103], 0.0103))	(([0.0085, 0.0106], 0.0074), ([0.0021, 0.004], 0.0082))	(([0.0072,0.0143], 0.0287), ([0.0072,0.0107,] 0.0107))
$\mathbb{P}_1$	(([0.0071, 0.0111], 0.0089), ([[0.0018, 0.0071], 0.0071))	(([0.0308,0.0359], 0.0051), ([0.0051,0.0103], 0.00205))	(([0.0063, 0.0095], 0.0063), ([0.0021, 0.004], 0.0084))	(([0.0072, 0.0143], 0.0143), ([0.0072, 0.0107], 0.0107))
$\mathbb{P}_2$	(([0.0071, 0.0125], 0.0175), ([[0.0071, 0.0089], 0.0071))	(([0.0308, 0.0359], 0.0051), ([0.0077, 0.0103], 0.0154))	(([0.0084, 0.0106], 0.0074), ([0.0042, 0.0061], 0.0082))	(([0.0072, 0.0107], 0.0287), ([0.0072, 0.0215], 0.0143))
$\mathbb{P}_3$	([([0.0036, 0.0107], 0.0036), ([0.0107, 0.0196], 0.0071))	(([0.0051, 0.0103], 0.0051), ([0.0087,0.0103], 0.0205))	(([0.0021, 0.0084], 0.0042), ([0.0042, 0.0084], 0.0087))	(([0.0072, 0.0107], 0.0143), ([0.0143, 0.0172], 0.0143))
$\mathbb{P}_4$	(([0.0071, 0.0075], 0.0075), ([0.0054, 0.0071], 0.0064))	(([0.0103, 0.0359], 0.0051), ([0.0051, 0.0103], 0.0103))	(([0.0032, 0.0095], 0.0042), ([0.0021, 0.0047], 0.0085))	(([0.0036, 0.0107], 0.0215), ([0.0071, 0.0143], 0.0107))

Table 10: Weighted Normalized Decision Matrix

Source: Author's own

Step 5: The optimality value and accuracy value for each alternative is in Table 11.

Table 11: Optimality Function $\widetilde{Q}$

Alternatives	$\widetilde{Q}_i$	$\widetilde{P}_i$
Ideal best	([0.0536,0.0733], 0.0587, [0.0162,0.0321], 0, .0356)	0.05643
$\mathbb{P}_1$	([0.0515,0.0708], 0.0346, [0.0162,0.0321], 0, .0467)	0.042
$\mathbb{P}_2$	([0.0535,0.0697], 0.0587, [0.0262,0.0468], 0, .045)	0.0525
$\mathbb{P}_3$	([0.018,0.0401], 0.0272, [0.0379,0.0555], 0, .0506)	0.0174
$\mathbb{P}_4$	([0.0242,0.0636], 0.0383, [0.0197,0.0364], 0, .0359)	0.0356

Source: Author's own

Step 6: The utility degree for each alternative is shown in Table 12.

Table 12: Utility Degree and Ranking of Alternatives

Alternatives	$\widetilde{\boldsymbol{U}}_{\boldsymbol{i}}$	**Ranking**
$\mathbb{P}_1$	0.7443	2
$\mathbb{P}_2$	0.930	1
$\mathbb{P}_3$	0.3083	4
$\mathbb{P}_4$	0.6309	3

Source: Author's own

6. DISCUSSION AND COMPARATIVE ANALYSIS

The ICF-ARAS framework has theoretical benefits and practical advantages over current fuzzy MCDM approaches. Theoretical benefits include the use of ICFGs, which can effectively represent an intuitionistic type of membership function, values representing non-membership function(s), and the possibility of hesitation(s), and their relationships among the criteria and alternatives; this increases the ability to interpret the framework and understand its structural representation. Practically, the use of an ARAS-based utility measure provides greater transparency in the utility measure than that found in TOPSIS, as it utilizes distance-based methods. The ICF decision matrix also allows for the inclusion of uncertainty through interval-valued assessments. Alternative optimality and utility measurements are calculated through normalization and the aggregation of the suitable alternative weights.

The obtained utility values are $\widetilde{U}_i(\mathbb{P}_2) = 0.930$, $\widetilde{U}_i(\mathbb{P}_1) = 0.7443, \widetilde{U}_i(\mathbb{P}_4) = 0.6309$, $\widetilde{U}_i(\mathbb{P}_3) = 0.3083$. Therefore the Candidates are ranked as : $\mathbb{P}_2 \geq \mathbb{P}_1 \geq \mathbb{P}_4 \geq \mathbb{P}_3$.

This outcome shows that candidate ($\mathbb{P}_2$) has the best overall fit to the teaching job, primarily because of better performance in a variety of criteria when considered within an intuitionistic cubic fuzzy context. The rather lower value of ($\mathbb{P}_3$) indicates poorer consistency of performance and greater values of hesitation which prove the sensitivity of the offered model to the changes in both membership and non-membership levels. The findings affirm that the ICF-ARAS model can accurately identify slight differences amongst the options even with inaccurate assessments, which ensures that there is a steady prioritization mechanism employed during decision making in a highly uncertain environment.

To further examine the validity of the proposed approach, the same dataset was analyzed using two existing techniques, namely ICF-WASPAS (Senapati, et al.,

2021) and ICF-TOPSIS (Muneeza et al., 2021). All methods produced the identical ranking order $\mathbb{P}_2 \geq \mathbb{P}_1 \geq \mathbb{P}_4 \geq \mathbb{P}_3$. which confirms the stability and correctness of the proposed ICF-ARAS method.

The obtained results of the ranking are the same, but the proposed approach has a number of notable benefits. In contrast to ICF-TOPSIS where the distance is used to identify the proximity of the solutions to an ideal solution and an anti-ideal solution, the ARAS-based architecture assesses each alternative in a first place with the help of direct comparison with an ideal reference, which leads to a higher interpretability level of the utility. This property finds its application especially during the process of explaining decisions to the stakeholders. In addition, ICFARAS frameworks have a simpler computational structure with an equivalent strength of discrimination compared to ICF-WASPAS, which involves the hybridization of additive and multiplicative aggregation schemes. The comparative results are shown in Table 13 and Figure 6, which further emphasize that the previous techniques had the same ranking as our proposed technique. This shows that the proposed technique is practical, adaptable, and significant.

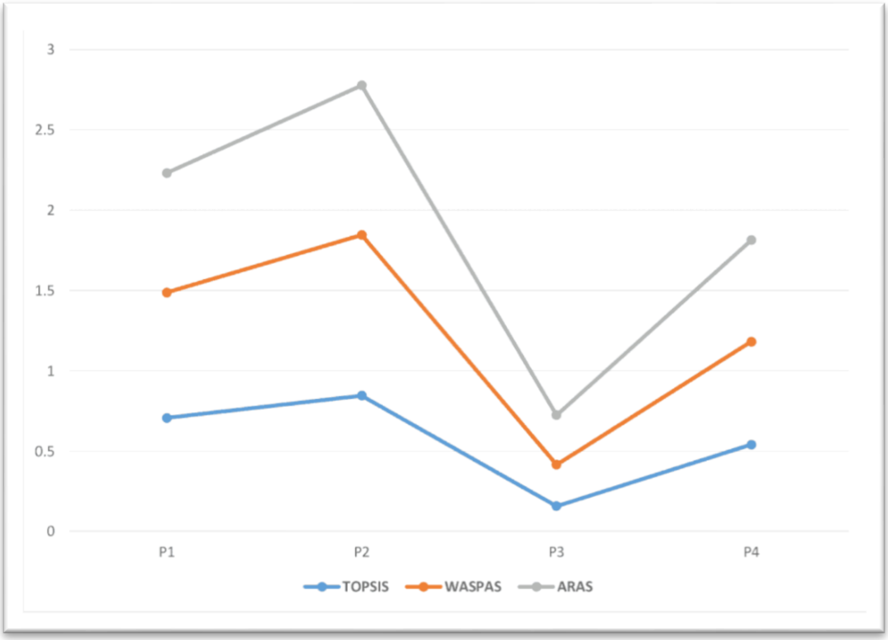
Table 13: Comparison Analysis

Alternatives	ICF-WASPAS		ICF-TOPSIS		ICF-ARAS	
	Score	Ranking	Score	Ranking	Score	Ranking
$\mathbb{P}_1$	0.7072	2	0.7803	2	0.7443	2
$\mathbb{P}_2$	0.8459	1	1.000	1	0.930	1
$\mathbb{P}_3$	0.1576	4	0.2585	4	0.3083	4
$\mathbb{P}_4$	0.5411	3	0.6418	3	0.6309	3

Source: Author’s own

Lastly, using ICFGs helps the proposed structure to ensure that the relationship between criteria and candidates is maintained, as most of the conventional cubic fuzzy MCDM ignores these relationships. Such structural modelling has important descriptive and analytical strengths of the framework. Therefore, the comparative analysis confirms that the suggested ICF-ARAS method is compatible with the already existing methods and even more interpretable, structurally expressive, and efficient in calculation, which makes it a strong instrument of complex real-world problems of decision-making. Here is the full, reviewer friendly, and similarity-free conclusion section to your paper on ICF-ARAS over ICFGs.

Figure 6: Comparison of ICF-ARAS Technique with other Techniques



Source: Author's own

7. CONCLUSION

For detecting occupational fraud, organizations use various techniques, tools, and procedures. These include internal audits, external audits, internal control systems, forensic audits, etc. Whistleblowing is considered one of the most effective tools for fraud detection. A whistleblowing system consists of several components, such as anonymous reporting channels (ARC), job security (JS) for whistleblowers, previous outcomes of reported whistleblowing events (PWB), and whistleblowing incentives (WBI). These attributes play a significant role in making the system more effective and result oriented.

This paper presented a new decision-making model, which is founded on combining an ICFG with the ARAS technique to deal with uncertainty, hesitation, and vagueness related to the real-world needs of MCDM. The intuitionistic cubic structure, in contrast to classical fuzzy and intuitionistic fuzzy models, simultaneously represents degree of interval-valued membership, non-membership, and degree of precise hesitation under intuitionistic cubic nature, thus a more detailed and adaptable mathematical summary of human judgments.

The ability and applicability of the intended ICF-ARAS approach was demonstrated by a realistic case study involving faculty recruitment in which four candidates were compared based on various qualitative parameters. The framework maintained the structural dependencies between decision elements by modeling the candidate and criteria as nodes in an ICFG and expressing the relationships between them as intuitionistic cubic fuzzy edge weights. The acquired ranking ($\mathbb{P}_2 \geq \mathbb{P}_1 \geq \mathbb{P}_4 \geq \mathbb{P}_3$) evidenced that the suggested method can provide consistent and interpretable results even when ambiguous and partially credible information is introduced. A comparative study with the already existing methods, i.e. ICF-WASPAS and ICF-TOPSIS shared the same ordering results, thus affirming the accuracy and consistency of the proposed model. Nevertheless, ICF-ARAS approach also has more benefits related to computational simplicity, clear aggregation of preference data, and the potential to retain graph-based relational knowledge, which is not straightforwardly considered in the majority of cubic fuzzy MCDM models.

In practical terms, the suggested model is highly flexible and can be successfully implemented in a range of areas of application, such as personnel selection, supplier assessment, healthcare diagnostics, project prioritization, and risk evaluation. The inherent interdependence of decision criteria and the imprecision of evaluations in these situations makes the model especially relevant as future studies aim to design dynamic ICFG-based ARAS models that support time-varying preferences, the introduction of group decision-making logic that considers the opinions of heterogeneous experts, and the consideration of hybrid extensions that allow entropy-based or optimization-based determination of weights. Furthermore, the theoretical properties of the ICFG operators and their effects on the consistency of the decision are promising directions of future research. The expected impacts of these extensions are that it will expand the usefulness of the proposed framework and increase its usefulness in large and complicated decision situations.

The in-depth case of this article only includes few candidates and criteria, which might limit the generalizability. But the main purpose is to prove the applicability and validity of the proposed ICF-ARAS model rather than empirical generalization. Future research could also use this framework with bigger datasets, multiple departments, or at diverse institutions to test its efficacy.

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